

2016

## MATHEMATICS — HONOURS

Eighth Paper

(Module – XV)

Full Marks – 50

*The figures in the margin indicate full marks**Candidates are required to give their answers in their own words as far as practicable*

Group – A

(Marks – 25)

Answer *any five* questions

1. (a) Prove that  $\mu \equiv \sqrt{1 + \frac{1}{4} \delta^2}$ .

where the symbols have their usual meanings. 3

(b) Prove that  $\Delta \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}\left(\frac{1}{2x^2}\right)$ , for  $h = 1$ . 2

2. If  $f_i$  is the value of  $f(x)$  at  $x = x_i$ , where  $x_i = x_0 + ih$  ( $i = 1, 2, 3, \dots$ ) and  $h > 0$ , prove that

$$f_i = E^i f_0 = \sum_{j=0}^i \binom{i}{j} \Delta^j f_0,$$

where  $\Delta$  and  $E$  be the forward difference and shift operators respectively. 53. Define  $n$ th order divided difference of a function  $f(x)$ .If  $f(x) = (x - x_0)(x - x_1) \dots (x - x_n)$  prove that $f(x_0, x_1, x_2, \dots, x_n, x, t) = 0$  for all  $x$  and  $t$  where the symbols have their usual meaning. What is meant by confluent divided difference? 1+3+14. Find the quadratic polynomial which takes the same values as  $f(x)$  at  $x = -1, 0$  and  $1$ . Integrate it to obtain

$$\int_{-1}^1 f(x) dx = \frac{1}{3} [f(-1) + 4f(0) + f(1)]$$

Assuming that the truncation error to have the form  $Af^{IV}(\xi)$ ,  $-1 < \xi < 1$ ,show that  $|A| = \frac{1}{90}$ . 1+2+25. For finding the square root of  $a$  ( $a > 0$ ), derive the iteration formula $x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right)$ ,  $n = 0, 1, 2, \dots$ , where  $x_0$  is any initial approximation and  $x_n$  is the  $n$ th approximate of the root. Find the order of convergence of the method. 3+2

6. Discuss how the inverse of a non singular matrix can be determined by using Gauss elimination method. What is the importance of pivoting in Gauss elimination method? 4+1

7. Explain Regula Falsi method of computing a simple real root of an equation  $f(x) = 0$ . Give the geometrical significance of this method. 4+1

8. What do you mean by eigen pair? Explain the power method for finding dominant eigen pair. 1+4

[Turn Over]

9. Given  $\frac{dy}{dx} = 1 - y^2/x$ , where  $y(2) = 2$ . Compute  $y(2.1)$  by Euler's Modified method, correct to four decimal places, taking  $h = 0.05$ .

5

**Group - B**

(Marks - 25)

**Section - I**

Answer *any two* questions

10. (a) Convert  $x = (567.425)_8$  into hexadecimal form and  $y = (EF.A1)_{16}$  into octal form.

2

(b) Obtain the binary equivalent of the following decimal numbers :

$$a = (22.625)_{10}$$

$$b = (14.25)_{10}$$

Compute  $a - b$  using 1's complement method.

3

(c) What is a flowchart? Draw a flowchart for sorting the following list of real numbers in ascending order : 91, 34, 28, 10.

4

11. (a) Write a C or FORTRAN program to test whether a given year is a leap year or not.

4

(b) Write an efficient FORTRAN 77/90 or C program to compute the value of  $f(x)$  for  $x = 0(1)10$  when  $f(x)$  is defined as

$$f(x) = x^2 \cos x \quad 0 \leq x \leq 2$$

$$= xe^x + 1 \quad 2 < x \leq 10$$

$$= \log_{10}(4 + 2x) \quad x > 10$$

5

12. (a) Given the lengths of three line segments  $a$ ,  $b$  and  $c$ ; write an efficient FORTRAN 77/90 or C program to test whether the line segments form a triangle. In case they form a triangle test whether the triangle is obtuse angled, right angled or acute angled.

5

(b) Write an efficient program in C or in FORTRAN to display the odd and even integers separately for the integers between 1 and 100.

4

13. (a) Write an efficient program in C or in FORTRAN to find a root of the equation  $e^x - 3x = 0$ , correct to six decimal places using the method of bisection in the interval  $[0, 1]$ .

5

(b) Write an algorithm to find a real root of  $f(x) = 0$  using fixed point iteration method.

4

**Section - II**

Answer *any one* question

14. (a)  $(a+b).(b+c).(c+a) = a.b + b.c + c.a$ , where  $a, b, c$  are any three elements of Boolean algebra  $(B, +, \cdot, ')$ .

2

(b) Let  $S$  be the set of all positive divisors of 30. Define the binary operations  $+$  and  $\cdot$  on  $S$  by  $a+b = \text{l.c.m. of } a, b$ ;  $a \cdot b = \text{g.c.d. of } a, b$  and  $a' = \frac{30}{a}$ . Show that  $(S, +, \cdot, ')$  is a Boolean Algebra.

5

15. (a) Find the disjunctive normal form of  $(x + y + z)(xy + x'z)$ .

3

(b) Design a circuit so that a light will glow when a majority of votes is cast in favour of the proposal from four voters who have voting weights of 4, 3, 1, 1 respectively.

4