2016

PHYSICS — HONOURS

Second Paper

(Group – A)

Full Marks – 50

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Answer ***Question No. 1*** and ***any four*** from the rest

1. Answer ***any five*** of the following : 2×5

(a) Find the velocity of the particle, given its position $\bar{r}(t) = A\left(e^{\epsilon t}\,\hat{i} + e^{-\epsilon t}\,\hat{j}\right)$. What angle does the velocity subtend with position vector ?

(b) Find the centre of mass of a thin homogeneous semi-circular disc of radius a and surface mass density σ.

(c) The power output of a certain horse is about 1 kW when putting a cart with a force of 400 N. Estimate the velocity of the cart.

(d) Determine the ratio of two heat capacity (γ) of a gaseous system containing (relaxed) diatomic molecules.

(e) Estimate the thickness of brick required to provide the same insulation as pine wood. Given thermal conductivities of brick and pine wood are 0.6 and 0.13 W/(m.°C) respectively.

(f) The intensity vs. wavelength of a black body is found to have its maximum at wavelength, $\lambda_m = 0.2$ cm and total energy radiated per second, J is measured to be 30 W. If the temperature of the black body is now doubled, find the new value of λ_m and J.

2. (a) A table tennis ball is released close to the surface of the moon, where there is no atmosphere (hence, no air drag), with an initial velocity $\bar{v}_0 = (0, 5, -3)$ m/s. It accelerates downward uniformly with acceleration $\bar{a} = (0, 0, -1.6)$ m/s^2. Find its velocity and position after 5 s. You may assume the initial position to be (0, 0, 0).

(b) For a particle moving along the line y = 2 with constant velocity $\bar{v} = u\,\hat{i}$, express $\bar{v}$ in plane polar coordinates. (3+3)+4

3. (a) Two particles of mass m_1 and m_2 move under mutual interacting force (central). Set up the equation of motion of the system and find an expression for the reduced mass.

(b) A system of particles with masses m_i and position vectors $\bar{r}_i$ (i = 1, 2, . . . , n) moves under its own mutual gravitational attraction alone.

[*Turn Over*]

Write down the equation of motion for $\bar{r}_i$. Show that a possible solution of the equation of motion is given by $\bar{r}_i = t^{2/3}\,\bar{a}_i$ where $\bar{a}_i$'s are constant vectors satisfying

$$\bar{a}_i = \frac{9G}{2}\sum_{j\neq i}^{n} \frac{m_j\left(\bar{a}_i - \bar{a}_j\right)}{\left|\bar{a}_i - \bar{a}_j\right|^3}$$

where G is the Gravitational constant. Show that for this system, the total angular momentum about the origin and the total linear momentum both vanish. What is the angular momentum about any other fixed point ? 3+(1+2+2+2)

4. Consider a thin homogeneous sphere of side a and total mass M.

(a) Using symmetry considerations, calculate the tensor of inertia.

(b) Determine the eigenvalues of the tensor of inertia obtained above. Hence, find the principal axes of inertia.

(c) Determine the ellipsoid of inertia for rotation about the origin and the moments of inertia for rotation about (i) x-axis (ii) y-axis. 3+4+3

5. (a) Consider an ideal gas consisting of a large number N of molecules inside a container of volume V at a temperature T. The gas molecules collide elastically with each other and the walls of the container. Using the principles of conservation of energy and momentum to model collisions between the gas molecules and the walls of the container, find an expression for the pressure of the gas in terms of volume V, particle number N and temperature T.

(b) Show that the fraction of gas molecules whose speeds differ by less than 1% from the value of the most probable speed (i.e. $C_m - 0.01\ C_m < c < C_m + 0.01\ C_m$ or $c - C_m < 0.01\ C_m$) is about 1.66%. 7+3

6. (a) Find out a relation between the coefficient of viscosity and thermal conductivity of an ideal gas on the basis of kinetic theory of gas.

(b) Express the van der Waals' equation of state as a virial equation and hence, find an expression for the Boyle temperature. 6+4

7. (a) Taking radiation loss into account, obtain Fourier's heat equation for one dimensional flow.

(b) Thermal conductivity of ice is $2.10\ \mathrm{W\,m^{-1}\,K^{-1}}$. How much time is required for a layer of ice of density $900\ \mathrm{kg\,m^{-3}}$ to become double in thickness when the ambient temperature is $-20^{\circ}\mathrm{C}$? Latent heat of ice is 3.36×10^5 W/kg.

(c) Assuming the sun as black body, estimate the temperature of the sun from the following data :

Angular diameter of the sun from the earth = 32 min, solar constant = $1356\ \mathrm{W\,m^{-2}}$, Stefan constant $\sigma = 5.67\times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$. 4+3+3